PROCEEDINGS

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SIR Wm. R. HAMILTON, LL.D., President, in the Chair.

Mr. Clarke read a supplement to his paper "on Atmospheric Electricity."

The author gave in this supplement a more detailed description than he had before done of the mode of insulating the apparatus for experiments on atmospheric electricity, which he had used in the course of his recent researches.

He then described an experiment by which he had shown the absence of decomposing agency in the electricity of serene weather, and stated his opinion of the cause.

Mr. Clarke next directed attention to the fact, that the curve representing the diurnal variation of the barometric column was the reverse of the electric, thermometric, and hygrometric curves. He considered that such a result was to be expected; for the barometric column should naturally be lower from mid-day to 3 P. M. than at midnight, in consequence of the greater quantity of aqueous vapour which exists in the atmosphere at the former than at the latter time,—air charged with aqueous vapour being known to be

of less specific gravity than dry air. Thus the barometric and hygrometric curves would be the inverse of each other, the maxima of the one corresponding to the minima of the other; and as the author had previously shown that the hygrometric, thermometric, and electrometric curves were in accordance, the barometric curve would be the inverse of the thermometric and electrometric curves also. The author remarked, that if this character of the horary oscillations of the barometer in Ireland be confirmed by the experiments of other observers, it will either lead to new views of this phenomenon generally, or show that the quantity of aqueous vapour existing in Ireland is so great as to cause the horary barometric oscillations to present themselves in a different form from that in which they are recognised in drier climates.

The author adverted, in the last place, to the hypothesis of Priestley and Beccaria,—that the upper regions of our atmosphere were the chief depositories of the electric fluid,—an opinion which he conceived must fall, if the origin of atmospheric electricity be due (as his experiments prove) to the existence of vapour; as these elevated parts of our atmosphere are far above the region of permanent vapour, or even of vapour at all.

Professor Mac Cullagh read a paper "on the Dynamical Theory of crystalline Reflexion and Refraction."

In a former paper, presented to the Academy in January 1837, and printed in volume xviii. of the Transactions, the author had reduced all the complicated phenomena of reflexion and refraction at the surfaces of crystals to the utmost regularity and order, by means of a simple rule, comprised in his theorem of the *polar plane*. This rule, which was verified by its agreement with exact experiments, he had deduced from a set of hypotheses relative to the vibrations of light in their passage through a given

medium, and out of one medium into another; but he had not attempted to account for his hypotheses, nor to connect them together by any known principles of mechanics; and the only evidence in favour of their truth, was the truth of the results to which they led. He had observed, however, that these hypotheses were not independent of each other; he had ascertained that the laws of reflexion at the surface of a crystal were connected with the laws of propagation in its interior; and he had thence been led to conclude that all these laws and hypotheses "had a common source in other and more intimate laws not yet discovered." He became impressed, in short, with the idea "that the next step in physical optics would lead to those higher and more elementary principles by which the laws of reflexion and the laws of propagation are linked together as parts of the same system."

This step the author has now made; and the present paper realises the anticipations scattered through the former. Setting out with the general dynamical theorem expressed by the equation

$$\iiint dxdydz\left(\frac{d^2\xi}{dt^2}\delta\xi+\frac{d^2\eta}{dt^2}\delta\eta+\frac{d^2\zeta}{dt^2}\delta\zeta\right)=\iiint dxdydz\delta\mathrm{v}, \quad (1)$$

where ξ, η, ζ, are the displacements at the time t of a particle whose co-ordinates are x, y, z, and where the density of the ether is supposed to be unity, as being constant for all media, the author determines the form of the function v, for the particular case of luminiferous vibrations, by means of the property which may be regarded as distinguishing them from all others—namely, that they take place entirely in the surface of the wave. From this property he shows, in the first place, that v is a function of the three differences

$$\frac{d\eta}{dz}-\frac{d\zeta}{dy}, \quad \frac{d\zeta}{dx}-\frac{d\xi}{dz}, \quad \frac{d\xi}{dy}-\frac{d\eta}{dx};$$

and, in the next place, that the only part of it which comes into play is of the second order, containing the squares and products of those quantities, with of course six constant coefficients. Then, supposing the axes of coordinates to be changed, he proves that the usual formulæ for the transformation of coordinates apply also to the transformation of those differences; so that, by assuming the new axes properly, the terms in the function v which depend on the products of the differences may be made to vanish, and v will then contain only the three squares, each multiplied by a constant coefficient. The axes of coordinates in this position are defined to be the principal axes, (commonly called the axes of elasticity); and when we put, with reference to these axes,

$$-2\text{v} = a^2\left(\frac{d\eta}{dz}-\frac{d\zeta}{dy}\right)^2 + b^2\left(\frac{d\zeta}{dx}-\frac{d\xi}{dz}\right)^2 + c^2\left(\frac{d\xi}{dy}-\frac{d\eta}{dx}\right)^2, \quad (2)$$

it turns out that a, b, c, are the three principal velocities of propagation within the crystal.

To find the laws of propagation in a continuous medium of indefinite extent, we have only to take the variation of v from the expression (2), and, after substituting it in the right-hand member of equation (1), to integrate by parts, so as to get rid of the differential coefficients of the variations $\delta\xi$, $\delta\eta$, $\delta\zeta$. Then equating the quantities by which these variations are respectively multiplied in the triple integrals on each side of the equation, we obtain the value of the force acting on each particle in directions parallel to the principal axes. The double integrals which remain on the right-hand side of the equation are to be neglected, as they belong to the limits which are infinitely distant. The resolved values of the force thus obtained lead to the precise laws of double refraction which were discovered by Fresnel, with this difference only, that the vibrations come out to be *parallel* to the plane of polarisation, whereas he supposed them to be *perpendicular* to it.

When there are two contiguous media, and the light passes out of one into the other, suppose out of an ordinary into an extraordinary one, and we wish to determine the laws of the reflected and refracted vibrations, it is only necessary to attend to the double integrals in the equation of limits; but the integrations must now be performed with respect to other coordinates. Taking the separating surface of the two media for the new plane of xy, the axis of x being in the plane of incidence, let the principal axis x of the crystal make with these new axes the angles α, β, γ, while the principal axes y and z, in like manner, make with them the angles α', β', γ', and $\alpha'', \beta'', \gamma''$, respectively. Then, marking with accents the quantities relative to the new coordinates, we have

$$\left.\begin{aligned}
\frac{d\eta}{dz}-\frac{d\zeta}{dy} &= \left(\frac{d\eta'}{dz'}-\frac{d\zeta'}{dy'}\right)\cos\alpha+\left(\frac{d\zeta'}{dx'}-\frac{d\xi'}{dz'}\right)\cos\beta \\
&\quad +\left(\frac{d\xi'}{dy'}-\frac{d\eta'}{dx'}\right)\cos\gamma, \\
\frac{d\zeta}{dx}-\frac{d\xi}{dz} &= \left(\frac{d\eta'}{dz'}-\frac{d\zeta'}{dy'}\right)\cos\alpha'+\left(\frac{d\zeta'}{dx}-\frac{d\xi'}{dz'}\right)\cos\beta' \\
&\quad +\left(\frac{d\xi'}{dy'}-\frac{d\eta'}{dx'}\right)\cos\gamma', \\
\frac{d\xi}{dy}-\frac{d\eta}{dx} &= \left(\frac{d\eta'}{dz'}-\frac{d\zeta'}{dy'}\right)\cos\alpha''+\left(\frac{d\zeta'}{dx'}+\frac{d\xi'}{dz'}\right)\cos\beta'' \\
&\quad +\left(\frac{d\xi'}{dy'}-\frac{d\eta'}{dx'}\right)\cos\gamma''.
\end{aligned}\right\} \quad (3)$$

Now if we take the variations of these expressions, and substitute them in the value of δv derived from equation (2), then multiply by $dx'dy'dz'$, and integrate between the limits $z'=0$ and $z'=\infty$, neglecting to take account of the latter limit, as well as of the integrations with respect to x' and y', of which both the limits are infinite, we shall get, in the equation which holds at the separating surface, a term of the form

$$\iint dx'dy'(\text{Q}\delta\xi' - \text{P}\delta\eta'), \tag{4}$$

where

$$\left.\begin{aligned} \text{P} &= a^2\left(\frac{d\eta}{dz} - \frac{d\zeta}{dy}\right)\cos\alpha + b^2\left(\frac{d\zeta}{dx} - \frac{d\xi}{dz}\right)\cos\alpha' \\ &\quad + c^2\left(\frac{d\xi}{dy} - \frac{d\eta}{dx}\right)\cos\alpha'', \\ \text{Q} &= a^2\left(\frac{d\eta}{dz} - \frac{d\zeta}{dy}\right)\cos\beta + b^2\left(\frac{d\zeta}{dx} - \frac{d\xi}{dz}\right)\cos\beta' \\ &\quad + c^2\left(\frac{d\xi}{dy} - \frac{d\eta}{dx}\right)\cos\beta''. \end{aligned}\right\} \tag{5}$$

This term, along with a similar but simpler one arising from the ordinary medium, must be equal to zero ; and as the variations $\delta\xi'$ and $\delta\eta'$ are independent, this condition is equivalent to two. Moreover, the quantities ξ' and η' are to be put equal to the corresponding quantities in the other medium, and thus we have two more conditions, which are all that are necessary for the solution of the problem.

The four conditions may be stated by saying, that each of the quantities P, Q, ξ', η' retains its value in passing out of one medium into another. Hence it is easy to show that the *vis viva* is preserved, and that ζ' likewise retains its value. These two consequences were used as hypotheses by the author in his former paper, and accordingly all the conclusions which he has drawn in that paper will follow from the present theory also.

It will be perceived that this theory employs the general processes of analytical mechanics, as delivered by Lagrange. The first attempt to treat the subject of reflexion and refraction in this manner was made by Mr. Green, in a very remarkable paper, printed in the Cambridge Transactions, vol. vii. part 1. After stating the dynamical principle expressed by equation (1), (though with a different hypothesis respecting the density of the ether,) Mr. Green ob-

serves, that, supposing the function v to be known, "we can immediately apply the general method given in the *Mécanique Analytique,* and which appears to be more especially applicable to problems that relate to the motions of systems composed of an immense number of particles mutually acting upon each other." Such is certainly the great advantage of starting with that general principle; but the chief difficulty attending it, namely, the determination of the function v, on which the success of the investigation essentially depends, has not been surmounted by Mr. Green, who has consequently been led to very erroneous results, even in the simple case of *uncrystallized* media, to which his researches are exclusively confined. In this case Mr. Mac Cullagh's theory confirms the well-known formulæ of Fresnel, one of which Mr. Green conceives to be inaccurate, and proposes to replace by a result of his own, which, however, will not bear to be tested numerically. The present theory applies with equal facility to all media, whether crystallized or not, and is distinguished throughout by the singular elegance and simplicity of its analytical details; a circumstance which the author regards as a strong indication of its truth.

Mr. Lloyd exhibited to the meeting a specimen of a remarkable substance recently found in the principality of Carolath, in Silesia. It formed part of a cloth of 200 square feet in surface now in the possession of the King of Prussia. No description of this substance has yet been published; but Major Sabine and Mr. Lloyd were informed by Baron Humboldt (by whom the present specimen was kindly given) that M. Ehrenberg had examined it microscopically, and had found it to be an organic substance, consisting partly of vegetable and partly of animal matter;—the vegetable component being the *conferva rivularis,* the animal different species of Infusoria, of the family known by the name of *Bacillaria.*

To illustrate the origin of this substance, Mr. Lloyd

read the following note from Major Sabine, respecting a similar body which has been examined and described by M. Ehrenberg.

"In the year 1686, some workmen who had been fetching water from a pond seven German miles from Memel, on returning to their work after dinner, (during which there had been a snow storm,) found the flat ground around the pond covered with a coal-black, leafy, or paper-like mass; and a person, who lived near, said he had seen it fall like flakes with the snow. On examination, some of the pieces were found to be as large as a table, and were lying upon each other to the depth of the thickness of a finger. The mass was damp and smelt disagreeably, like rotten seaweed; but when dried the smell went off. It tore fibrously like paper. Specimens were preserved in several collections, where it was known by the name of *Meteor-paper*, and by many was actually supposed to be a meteoric body. It has been recently examined by M. Ehrenberg, and found to consist partly of vegetable matter, chiefly *conferva crispata,* (common in Germany,) and partly of *infusoria,* of which M. Ehrenberg was able to recognise twenty-nine species. Of these, eight species have siliceous coverings, but the others, which are equally well preserved, were soft-skinned animals; most of them are known as species now existing.

"The Meteor-paper, therefore, as it has been called, was formed in marshy places; had been raised into the air by storms of wind; and had again fallen.

"Substances of the same nature have been found in Norway, in Silesia, and in the Erz Mountains. In some instances they are described as *leathery;* in others as resembling *wadding,* and being white on the upper side and green beneath. They have probably all a similar origin."

Mr. Lloyd also laid on the table of the Academy a specimen of a very similar substance, which he had received

from Sir John Herschel, and which was found investing the rocks at the mouth of one of the rivers of Southern Africa. It resembles the other very much in external appearance, except that the fibres are coarser, and more compactly matted together. It appears to consist almost entirely of *confervæ*, but apparently of a different species.

A paper was read by Mr. J. Huband Smith, descriptive of certain porcelain seals, amounting to upwards of a dozen, found in Ireland within the last six or seven years, and in places very distant from each other.

He exhibited to the Academy one of these seals, with impressions of several others in sealing-wax. He stated that they were all uniform, consisting of an exact cube, having, by way of handle, some animal (probably an ape) seated upon it; and that they were so precisely similar in size and general appearance as to be undistinguishable, except by the characters on the under surface. Little is known respecting these seals beyond the mere fact of their having been found in this country.

An extract from the Chinese grammar of Abel-Rémusat showed that the inscriptions on these seals are those of a very ancient class of Chinese characters, "in use since the time of Confucius," who is supposed to have flourished "in the middle of the sixth century, before J. C." The remote period to which these characters are assigned, leaves open a wide field for conjecture as to the time in which these porcelain seals found their way into this country.

The situations in which some of them have been found are remarkable. One was discovered in ploughing a field near Burrisokane, county of Tipperary, in 1832; another was found last year at Killead, in the county of Down; another in the bed of the river Boyne, near Clonard, in the county of Meath, in raising gravel; and a fourth was discovered many years ago at a short distance from Dublin.

From the extreme degree of heat to which they appear to have been subjected, and the consequent vitrification which has in some measure taken place, they are quite as capable of resisting the attacks of time as the glass and porcelain deities and ornaments found in the mummy cases of Egypt, and may have lain for an indefinite period beneath the surface of the earth. It is therefore, at least, possible that they may have arrived hither from the East, along with the weapons, ornaments, and other articles of commerce, which were brought to these islands by the ships of the great merchant-princes of antiquity, the Phœnicians, to whom our ports and harbours were well known.

Mr. Smith then called the attention of the Academy to the remarkable discovery, by Rosellini, Lord Prudhoe, and other recent travellers, of unquestionable Chinese vases in the tombs of Egypt. He read a passage from Davis's China, in which some of them were described; and also an extract from Wilkinson's Ancient Egyptians, from which it appeared that the number of Chinese vases found at Coptos, Thebes, and elsewhere, amounted to seven or eight, and that the inscriptions on them had been translated by Chinese scholars to mean, "The flower opens, and lo! another year," being a line from an ancient Chinese poem.

From this the trade of China with distant countries, at a period of the remotest antiquity, being clearly proved, Mr. Smith submitted to the Academy that a case of strong probability had been made out, that the porcelain seals found their way into Ireland at some very distant period. In fact, if they be not of modern introduction into this country—a supposition which the situations in which several of them have been found seems utterly to preclude—their arrival here must of necessity have been most ancient.

Mr. Petrie read a paper "on ancient Seals of Irish Chiefs, and Persons of inferior Rank," preserved in the col-

lections of Irish Antiquities formed by the Dean of St. Patrick's, and by himself. He observed that this class of antiquities had been but little attended to by Irish antiquaries—a circumstance which he attributed to the want of general collections of our national antiquities till a recent period: and hence, if the question had been asked a short time since, whether the Irish had the use of signets generally amongst them or not, it would have been impossible to give a decisive answer. This question, however, can now be answered in the affirmative; but the period at which the use of seals commenced in Ireland is still uncertain, as no Irish seals anterior to the Anglo-Norman invasion have been found; or, if found, their discovery has not been recorded. As, however, it is now certain that seals were used by the Anglo-Saxons, it is not improbable that their use may have been introduced into Ireland also—more especially as a remarkable similarity prevailed between the two countries in customs and in knowledge of the arts.

The Irish seals hitherto discovered are similar in style and device to the cotemporary seals of the Anglo-Normans of similar ranks; and, like the secular seals of the latter, are usually of a circular form, whilst the ecclesiastical seals are usually oval.

The seals which Mr. Petrie described or exhibited to the meeting were as follows:—

1. A drawing of an impression from the seal of Felim O'Conor, King of Connaught, as published by Sir James Ware. The device exhibits the figure of Felim on horseback, charging with sword in hand, and the legend reads,

"S. FEDHLIM, REGIS CONACTIÆ."

This prince died in 1265.

2. The seal of Donald Og, the son of Donald Roe Mac Carthy, King or Prince of Desmond, who died in 1309. The device is similar to that of the preceding, and the legend reads,

"S. DONALDI OG FILI. D. ROGH MAC CARTHY."

This seal is in the collection of the author.

3. The seal of Mac Con, Chief of Hy-Caissin, a territory in Thomond, possessed by the family of Macnamara. He died about the year 1350. The device of this seal is also similar to that of Felim, and the legend reads,

"S. MICON DUCIS DE IV. CASSIN."

This seal is in the collection of the Dean of St. Patrick's.

4. The seal of Brian O'Brian, Prince of Thomond, who was killed in the year 1350. The device is a griffin, which appears to have been the heraldic badge of the O'Briens at this period; and the legend reads,

"S. BRIAN I BRIAN."

This seal is also in the collection of the Dean of St. Patrick's.

5. The seal of Murtagh O'Neill, who, as Mr. Petrie believes, was the Lord of Clannaboy of this name, whose death is recorded by the Irish annalists at the year 1471. The device is the bloody hand of O'Neill, and the legend reads,

"S. MAURITIUS UI NIELL."

This seal is also in the collection of the Dean of St. Patrick's.

6. The seal of Mac Craith, the son of O'Dafid. The device is a non-descript animal, and the legend reads,

"S. MAC CRAITH MAC I DAFID."

This seal, which is of the early part of the fourteenth century, appears to belong to the O'Daffy's, a family of the Dal Cais in Thomond, still in existence. The seal is in the collection of the author.

7. The seal of Brian O'Harny, chief of an ancient family in Kerry. The device exhibits the helmeted head of a warrior, cut on a cornelian, and the legend reads,

"S. BRIAN O'HARNY."

This seal is in the collection of the Dean of St. Patrick's.

The material of all these seals is silver.